AD P000631

DECISION RISK ANALYSIS
for
XM204, 105MM Howitzer, Towed
Reliability/Durability Requirements

Mr. Thomas N. Mazza
Mr. Robert C. Banash
US Army Armament Command

INTRODUCTION

There is a continuous discussion between the user and the designer as to what the reliability and durability requirements for a weapon system should be. This is particularly true for weapon systems which are primarily mechanical such as howitzers. The user documents a need (through the MN or ROC process) for a system possessing reliability and durability significantly higher than previous systems. The designer on the other hand feels the user should accept any system which is at least as good as the existing weapons reliability and durability, since the new design will undoubtedly possess other characteristics such as increased range, reduced weight, etc. which the designer feels are the primary reasons for the new system and are, in themselves, inversely related to reliability/durability. (He has never been asked to design a totally new system strictly to increase reliability or durability.) When the discussions are over and a compromise is reached, the true benefit of the agreed-to requirement to the Army is questionable. Each side attempts to provide enough documentation to support its position.

This analysis develops a rationale for the reliability and durability requirements for the XM204, 105MM Towed, Howitzer while simultaneously defining a plan to test for those requirements. The system reliability requirement, subsystem durability requirements, reliability and durability uncertainties of the proposed design, and the number of prototypes and test lengths to establish reliability and durability parameters, are related to expected costs.

Certain of these factors are identified as variables. This lends to consideration and evaluation of alternative courses of action with the objective of reducing expected life cycle costs. The expected loss (life cycle cost for this analysis) of an alternative is identified as the risk of that alternative in accordance with standard statistical terminology[1].

REQUIREMENTS

As a result of DT/OT-II decisions will be made as to the acceptability of the entire system from a reliability viewpoint and on each of the four major subsystems from a durability viewpoint. Therefore reliability requirements must be specified for the total system and durability requirements must be specified for each major subsystem. It was assumed that a truncated test would be preferred to a fire to failure test for planning purposes. Therefore, a maximum number of rounds to be fired or each system truncation point must be specified. As a total system configuration is

[1] Ferguson, T.S., Mathematical Statistics, A Decision Theoretic Approach, Academic Press, 1967.

Best
Available
Copy

required to conduct the test, the number of systems to be put on test must be specified along with the number of spare or replacement components. Also since statistical techniques produce not one but a family of alternative statements from the same test, the confidence level associated with the test must be specified. Additionally, each reliability and durability requirement must be specified. Rejection, fix and acceptance region were specified by the pairs (R_1,R_2) for reliability and (D_1,D_2) for durability (defined in the section "Loss Function"). Combining the above, the following set of requirements must be specified to define the requirements and statistical test environment for DT/OT-II.

System:

- Number of systems on test
- Reliability acceptance MTBF - R_1
- Reliability rejection MTBF - R_2
- Truncation Point - T_p
- Confidence Level

Subsystem:

- Number of spare subsystems - N
- Durability acceptance MTBF - D_1
- Durability rejection MTBF - D_2

The subsystem requirements must be specified for each major subsystem which are: the carriage, the recoil, the tube and the breech.

QUANTIFICATION

Research scientists and design engineers were interviewed to quantify their expectations regarding durability of the subsystems under their cognizance. Reliability expectations were developed by the WECOM Product Assurance Directorate based upon failure and stress data from the M102, 105MM Towed, Howitzer and expected stress levels and failure modes of the XM204.

The primary technique used to quantify the durability of the subsystem was presented by Stanford Research Institute at the 1972 US Army Operations Research Symposium. In essence, the design engineer is required to choose between two lotteries. Lottery No. 1 concerns the durability of the subsystem. The design engineer will win, say, one million dollars if the durability of the subsystem will be demonstrated less than X rounds (X is specified by the interviewer). Lottery No. 2 concerns the spin of a pointer on a wheel. The design engineer will win one million dollars if the pointer falls within the red sector. After a choice has been made by the interviewee, the red sector is increased or decreased with the object of making the interviewee indifferent between the lotteries. When the indifference has been obtained, the percentage of the exposed red sector is recorded as the belief of the interviewee in the occurrance of the event - subsystem durability is less than X rounds.

$$P[\text{durability} < X] = \%\ \text{red sector}$$

The process is repeated for various values of X until a probability distribution can be drawn. Two experts were interviewed for most of the major subsystems for which a durability requirement exists. The experts were either engineers working on the design of the subject subsystem or physical scientists with knowledge of the subsystem.

These data were input to the computer simulation in the form of a discrete distribution. The probability content of an interval was obtained (by subtraction of probability values at endpoints of the interval (and

assigned to the midpoint of the interval. These distributions are presented in Table 1 for the distribution fit to the data.

TABLE 1
PRIOR DISTRIBUTION ON MEAN-ROUNDS-TO-FAILURE PARAMETER
SUBSYSTEM

	CARRIAGE		RECOIL		TUBE		BREECH		RELIABILITY	
	MID-PT	PROB	MID-PT	PROB	MID-PT	PROB	MID-PT	PROB	MID-PT	PROB
1	2500	.05	2500	.24	4250	.32	48750	.03	1050	.10
2	7500	.05	7500	.14	4750	.16	56250	.07	1150	.15
3	12500	.05	12500	.13	5250	.12	63750	.40	1325	.15
4	17500	.04	17500	.15	5750	.10	71250	.30	1450	.10
5	22500	.04	22500	.15	6250	.10	78750	.13	1725	.15
6	27500	.04	27500	.16	6750	.08	86250	.07	2000	.10
7	32500	.08	32500	.03	7250	.06			2350	.10
8	37500	.15			7750	.06			2625	.05
9	47500	.25							3000	.05
10	52500	.25							3500	.05

The distribution quantify the uncertainty associated with the expected number of rounds to failure. The breech safe life and tube fatigue safe life were estimated to be one-third of this value. The expert opinion on the minimum safe life was higher than the optimistic estimates on tube wear life; this led to consideration of only tube wear in regard to estimating tube durability.

THE LOSS FUNCTION

The purpose of the loss function is to estimate the expected losses (expenditures which will occur when action is taken in accordance with the belief that the state of the system is S' when, in fact, it is S.

The contractually specified performance parameters, reliability (R) and durability (D), are considered to be bounded by military necessity or cost-effectiveness. From the military necessity standpoint, reliability can be translated into the requirement that a battery, fire on the average, a specified number of rounds during a mission. A system with a lower reliability will, on the average, fire fewer rounds. Increasing the number of systems per battery will achieve this goal of a minimum-expected-number-of-rounds/battery/mission. If the resulting design of the systems does not meet the specified limits, this alternative can be used as an upper bound on the cost of the second alternative, that being to "fix-up" a marginal system. In all cases an additional alternative is to cancel the program and live with the existing system. The term "fix-up" as used here means that a reliability growth program will be entered. A sequence of design-test cycles will be conducted until the reliability is grown to the required level

Similarly, durability is a requirement on the life of a system. Durability can be translated into the requirement that a system, on the average, survives a specified number of rounds before requiring an overhaul, or replacement when overhaul isn't applicable (i.e., tubes). A system with a lower durability will, on the average, survive fewer rounds before an overhaul is required. The cost of this lower than desired system durability

can be estimated by the expected increase in overhaul/maintenance actions, over a suitable time frame.

Reliability Loss Function, $L(R,R')$

Definitions:

R - true value of system reliability

$\hat{R}$ - statistical estimate of R based on test data

R' - $= R_2$ if $\hat{R}$ not significantly less than R_2 (based on statistical test of hypothesis)

$= \hat{R}$ if $\hat{R}$ is significantly less than R_2

R_1 - a value of R' which is less than or equal to R_1 is cause for system rejection

R_2 - a value of R' which is greater than or equal to R_2 is cause for system acceptance with regard to reliability. This value is viewed as a requirement designed to insure that the expected number of rounds fired by a battery in a particular mission will not be below a specified level.

$L(R,R')$ - is the costs incurred in taking a course of action when R is the true reliability and R' its estimate.

Consider a pair (R_1,R_2) to be defined such that if the true system reliability R were known, the following actions would occur (depending on R):

1. $R \leq R_1 \Rightarrow$ Action: Reject entire system

2. $R_1 < R < R_2 \Rightarrow$ Action: Fix - the system will be made acceptable, by entering a reliability growth program or fielding more systems per battery to insure the expected number of rounds criterion.

3. $R_2 < R \Rightarrow$ Action: Accept the system with respect to reliability.

Unfortunately, the value of R is not known. Statistical techniques will provide an estimate, $\hat{R}$, from test data. This value will be compared to R_2 to determine if $\hat{R}$ is significantly less than R_2 on a statistical basis. If the test does not show a significant difference then action will be taken as though $R' \geq R_2$, otherwise we will take action as though $R' = \hat{R}$.

Consider the reliability decision space divided into three regions as shown below.

rejection | fix | accept

R_1 R_2

The actual or true reliability, R, could fall into anyone of the three regions. In addition, when we test the system the estimate R' could also fall into anyone of the three regions. As we increase the sample size of our test R' should asymptotically approach R, however, the cost of the test will also increase. As we lower the test cost or reduce the sample size then the expected difference between R and R' will increase. Therefore, there are nine possible states that could occur. They are:

Case 1	RR'		
Case 2	R	R'	
Case 3	R		R'
Case 4	R'	R	
Case 5		R'R	
Case 6		R	R'
Case 7	R'		R
Case 8		R'	R
Case 9			R'R

The following discussion outlines a method for estimating the expected losses incurred for each of the three possible decisions when, in fact, R is the true system reliability. The nine cases as outlined above are grouped according to the decision that is made. Contained within the discussion of each case are several cost figures which are referred to as C_1, C_2, C_3, etc. The definition of each are as follows:

- C_1 - The cost of extending the life of the present (M102/M101A1) system during a new development program (6 years)
- C_2 - The cost of a new development program
- C_3 - Cost of procuring and operating a second generation design during the remaining planned life (14 years)
- C_4 - Cost of the planned first years procurement
- C_6 - Cost of a redesign effort to correct a R̲ failure mode
- C_7 - Cost to procure one XM204
- C_9 - Cost to operate and maintain one XM204 over 20 years

<u>Decision</u>:

Accept: | | RR' -- Case 9

Under this case the true system R is acceptable and as a result of the test the system is accepted. The correct decision is made and the only cost incurred are the cost to procure and the cost to operate the weapon over the 20-year life cycle. The cost of R̲ failures over the 20-years is based on the actual MRBF of the system.

$$L_9(R,R') = (C_7+C_9)(\text{No of Systems}) + (947.65)(\text{Total Rnds})/(\text{MRBF})$$

Accept: | R | R' - Case 6

Under this case the true system R lies within the fixup region and as a result of the test the system is accepted. An incorrect decision was made and the cost associated with this decision are as follows. Since it is thought that the system is good we go ahead with the first years production. However, after the first years production it is assumed that it will now be discovered that the true R is not as good as thought. A product improvement program is initiated and the system R is grown via a redesign-test cycle until the true system R is acceptable. Now since one years production has already been made a retrofit program will be needed. To cost this out it was assumed that it would cost a factor of two times the cost of an ordinary R growth program had it been determined (i.e., the right decision made) before the first years production was made, that the true R was not acceptable.

$L_6(R,R') = (C_7+C_9)(\text{No of Systems}) + (2)(\underline{R}\text{ growth cost})$

Accept: R —————— R' - Case 3

Under this case the rue system R is definitely not acceptable, but as a result of the test the system is accepted. An incorrect decision was made and the cost associated with it are as follows. Since it is thought that the system is good we go ahead with the first years production. It is assumed that it will now be discovered that the true system R is definitely unacceptable, and the total system will be rejected. The cost of the first years production will be lost and a new development program will be initiated. The present system will have to be maintained and operated during the new development program which is assumed to last six years, per AR 1000-1.

$$L_3(R,R') = C_1 + C_2 + (C_3)(\text{No of Systems}) + C_4$$

Note: It is assumed that as a result of the new development program the new system will meet the specified MN requirements - This applies to all cases where a new development program is entered.

Reject: RR' —————— - Case 1

Under this case the true system R is unacceptable and as a result of the test the system is rejected. The correct decision was made. A new development program will be entered and the life of the present system will be extended. In addition, the cost of the prototypes and test cost for the first design will be lost.

$$L_1(R,R') = C_1 + C_2 + (C_3)(\text{No of Systems}) + \text{Cost of Prototypes} + \text{Test Cost}$$

Reject: R' —— R —— - Case 4

Under this case the true system R lies in the fixup region. As a result of the test the system is rejected. Therefore the cost described for Case 1 are incurred.

$$L_4(R,R') = C_1 + C_2 + (C_3)(\text{No of Systems}) + \text{Cost of Prototypes} + \text{Test Cost}$$

Reject: R' —————— R - Case 7

Under this case the true system R is acceptable. As a result of the test the system is rejected. Therefore the cost described for Case 1 are incurred.

$$L_7(R,R') = C_1 + C_2 + (C_3)(\text{No of Systems}) + \text{Cost of Prototypes} + \text{Test Cost}$$

Fixup: —— R' —— R - Case 8

Under this case the true system R is acceptable. As a result of the test a R growth program is initiated. Funds will be allocated based on R'.

It should soon be learned that the true R is acceptable, but since the funds have been allocated the growth program will continue. This will increase the true R which will lower the total life cycle R cost.

$$L_3 = (C_7+C_9)(\text{No of Systems}) + \text{Cost of } \underline{R} \text{ Growth Program}$$

Fixup: R R' - Case 2

Under this case the true R is unacceptable. As a result of the test a R growth program is initiated. The funds for the growth program will have been sunk and soon it will be realized that the system should be rejected. Consequently, a new development program will be started, and the cost of Case 1 will also be incurred.

$$L_2(R,R') = C_1 + C_2 + (C_3)(\text{No of Systems}) + \text{Cost of } \underline{R} \text{ Growth Program}$$

Durability Loss Function, L(D,D')

There are two basic differences between the reliability loss function and the durability loss function. The first is that there are durability requirements at the subsystem level while reliability requriements are only at the system level. The second is in the concept of fixing a marginal system for reliability vs. accepting an increased maintenance burden for durability.

Definitions:

D - true value of subsystem durability

$\hat{D}$ - estimate of subsystem D based on test

D_1 - a value of D' which is less than or equal to D_1 is cause for subsystem rejection

D_2 - a value of D' which is greater than or equal to D_2 is cause for subsystem acceptance with regard to durability

$D' = D_2$ if $\hat{D}$ not significantly less than D_2

$= \hat{D}$ if $\hat{D}$ is significantly less than D_2

For each subsystem a pair (D_1, D_2) will be defined such that if the true subsystem durability D were known the following actions would occur, (depending on the value of D).

1. $D \leq D_1 \Rightarrow$ Action: Reject subsystem

2. $D_1 \leq D \leq D_2 \Rightarrow$ Action: Fixup - The cost incurred to maintain the subsystem at D vs. D_2 will be used as an upperbound for the cost of this action.

3. $D_2 \leq D \Rightarrow$ Action: Accept subsystem, plan to maintain subsystem based on D_2 being the true durability.

However, the value of D is not known. Statistical techniques will provide an estimate $\hat{D}$ from test data. This value will be compared to D_2 to determine if $\hat{D}$ is significantly less than D_2 on a statistical basis. If the test does not show a significant difference then action will be taken as though $D' \geq D_2$, otherwise we will take action as though $D' = \hat{D}$.

Similar to the reliability decision space, the durability decision space is divided into three regions as shown below:

Reject	Accept/Marginal	Accept
	D_1	D_2

As with the reliability the true durability D could fall into anyone of the three regions as could the estimate D'. Therefore, there are nine possible states that could occur. There are:

	Reject	Accept/Marginal	Accept
Case 1	DD'		
Case 2	D	D'	
Case 3	D		D'
Case 4	D'	D	
Case 5		DD'	
Case 6		D	D'
Case 7	D'		D
Case 8		D'	D
Case 9			DD'

There are only three instances where the decision would be to reject the subsystem, namely Cases 1, 4 & 7. If any subsystem is rejected then the cost incurred are the same as those that would occur for a reliability rejection. A new development program will be entered and the life of the present system will be extended. In addition, the cost of the prototypes and the test cost for the first design will be lost.

$$L_{1,4,7}(D,D') = C_1 + C_2 + (C_3)(\text{No of Systems}) + \text{Cost of Prototypes} + \text{Test Cost}$$

In all other cases the subsystem will be accepted, however, the expected number of renewals E[N] (overhauls) will differ depending on the decision space. For Cases 2,3,6 & 9 the expected number of renewals will be calculated based on the true mean time between durability failure D. For Case 8 the estimate D' will be used to calculate the expected number of failures. And for Case 5 the minimum of D and D' will be used.

$$L_{2,3,5,6,8,9}(D,D') = (E[N])(\text{Cost/overhaul})(\text{No of Systems})$$

For Cases 2,3,6 & 9 the test estimate D' is the mean time between overhaul the subsystem is thought to have. Once the end item is fielded, the true durability D is the actual maintenance burden that will be exhibited, therefore, the expected number of renewal based on D is the true cost. It would have to be overhauled at D.

For Case 8 the planned overhaul time would be based on D' and since D > D' it will not be possible to take advantage of the full designed durability. Therefore, the E[N] is based on D'.

For Case 5 the calculation of E[N] is based on the Min(D,D'). If D > D' then D' will be used as for Case 8. If D' > D then D will be used as for Cases 2,3,6 & 9.

Total Loss

The total expected cost if the system is accepted, is the sum of the reliability and durability losses. However, if any subsystem is rejected for durability or if the system is rejected for reliability then a total

redesign stage is entered. It is assumed that no matter what magnitude of improvement is required during the redesign stage, when the "new" system is tested it will meet all MN requirements regardless of what level the requirements are set at. The expected number of durability and reliability failures for the "new" design are calculated on the basis of the MN requirements over the remaining 14 (20-6) years.

PARAMETER SPACE

The procedure adopted for pursuing the objective of the study was to search over the relevant variables and choose that combination which yields the lowest expected loss.

The system reliability requirement for the XM204, states a minimum acceptable average number of rounds between failure (MRBF). This requirement assumes MRBF to be constant during the operating life of the system. A constant MRBF will be assumed for this study with respect to reliability. Subsystem durability requirements are expressed in terms of a subsystem operating no less than a specific number of rounds with a specified probability; e.g.,

$$\text{Prob [Subsystem Life} > 500 \text{ rounds]} \geq .5$$

A direct search with acceleration was adopted for searching the parameter space for parameter vectors yielding lower expected losses. This routine makes steps on either side of the baseline to establish a direction for each of the parameters (variables in this context) and takes larger or smaller steps in the established direction (constrained by a specified number of step cuts), until not further improvement can be made in the objective function, which in this case is expected loss.

The initial baseline reliability/durability validation test plan and requirements are presented in Table 3. The test of hypothesis confidence level (Table 2, A6) pertains to the test conducted on the statistic under consideration. (i.e., test data is used to generate a statistic which estimates durability, say $\hat{D}$. Is D significantly different than the desired durability D_2?

TABLE 2

BASE LINE PARAMETERS

A. Test Parameters

1. No. Carriage Subsystems - 3
2. No. Recoil Subsystems - 3
3. No. Tubes - 12
4. No. Breech Subsystems - 3
5. Truncation Point - 22,500
6. Test-of-Hypothesis Confidence (Assumed) - 90%

B. Reliability/Durability Parameters

1. Reliability Rejection, R_1 = 1500
2. Reliability Acceptance, $\hat{R}_2 = R_1$
3. Durability Rejection, $(0, D_1)$
   a. D_1 (Carriage) = 22,500

TABLE 2 CONTINUED

b. D_1 (Recoil) = 22,500
c. D_1 (Tube) = 7,500
d. D_1 (Breech) = 22,500

4. Durability Acceptance, (D_2, ∞)

a. D_2 (Carriage) = D_1 (Carriage)
b. D_2 (Recoil) = D_1 (Recoil)
c. D_2 (Tube) = D_1 (Tube)
d. D_2 (Breech) = D_1 (Breech)

TEST PLAN

During DT/OT a certain number (N_1) of howitzers will be placed on test. For testing purposes the howitzer is composed of one critical subsystem (the carriage) and several major non-critical subsystems (recoils, tubes & breeches). Each howitzer will be fired until one of two events occur:

(1) a carriage durability failure occurs,
(2) a specified number of rounds, t_p, have been fired.

A maintenance support test package will accompany each weapon and among its contents will be N_k spare prototypes for each of the major non-critical subsystems (N_2 - Recoil, N_3 - Tube, N_4 - Breech).

A total system configuration is required to conduct the test, however with respect to probability of failure, each subsystem is assumed independent. During the course of the test as each non-critical subsystem durability failure occurs, the failure time is noted, and the failed subsystem is replaced until either:

(a) all of the spare prototypes of type k have suffered a durability failure,
(b) the carriage has suffered a durability failure or has fired t_p rounds.

If all of the spares of a particular type subsystem have failed before (1) or (2) above occur, then that subsystem will be "patched-up" to allow the test to continue until either (1) or (2) does occur. However, no additional information will be collected on that weapon for that subsystem.

When a reliability failure occurs for any subsystem the failure time is noted and the failure is repaired to allow the test to continue. The repair will be assumed as-good-as-new and each reliability failure is assumed independent.

A hypothetical design and observation of this type of test is shown in the following example:

Example 1:

Number of Carriages, N_1	3
Number of Recoils/carriage, N_2	4 (original + 3 spares)
Number of Tubes/carriage, N_3	7 (original + 6 spares)
Number of Breeches/carriage, N_4	2 (original + 1 spare)
t_p = 22,500 rounds	

The above test design depicts a test where three howitzers will be fired for a maximum of 22,500 rounds each. Each carriage has three spare recoils, six spare tubes, and one spare breech in its maintenance support test package.

Test Observations

	Reliability Failures	Durability Failures
Carriage #1	3085, 5667, 15594	15597
Recoil #1.1	8766, 10729	No observed failure
Tube #1.1		6648
Tube #1.2		8823
Tube #1.3		14402
Tube #1.4		No observed failure
Breech #1.1		No observed failure
Carriage #2	8020, 1[illegible]672	No observed failure
Recoil #2.1		9166
Recoil #2.2	13587, 18178	20293
Recoil #2.3		No observed failure
Tube #2.1		6822
Tube #2.2		13339
Tube #2.3		20122
Tube #2.4		No observed failure
Breech #2.1	22498	
Carriage #3	5552, 9229, 18178	No observed failure
Recoil #3.1	11443	11666
Recoil #3.2		16674
Recoil #3.3	22498	No observed failure
Tube #3.1		7270
Tube #3.2		17924
Tube #3.3		No observed failure
Breech #3.1		No observed failure

The above failure times are the number of rounds on the carriage at the time the failure occurred. Carriage #1 has a durability failure at 15597 rounds at which time all testing was stopped on that weapon. Testing on Carriage #2 and #3 were stopped at the predetermined truncation point of 22,500 rounds.

Associated with weapon #1 were five reliability failures which occurred at the times shown. Three of the reliability failures occurred on the carriage and two occurred on the recoil. The original recoil did not have a durability failure and lasted until the carriage failed or 15597 rounds. The original tube was replaced at 6648 rounds, the first spare was replaced 8823 rounds, and the second spare was replaced at 14402 rounds. The last tube did not fail in the remaining 1195 rounds. The original breech survived the 15597 rounds.

Test Statistics

The observations as outlined in Example 1 were generated from the two parameter Weibull distributions as shown below. Along with the true values are shown the estimates which are based on the observations.

	True	Estimate
Carriage		
Shape parameter	= 2.48642	$\hat{\alpha}$ = 3.15846
Scale parameter	= .449008 x 10^{-11}	$\hat{\lambda}$ = .775047 x 10^{-14}
with MTBF	= 31,613 rounds	$\hat{\mu}$ = 26,152 rounds
Recoil		
Shape parameter	= 1.21277	$\hat{\alpha}$ = 2.05138
Scale parameter	= .487712 x 10^{-5}	$\hat{\lambda}$ = .379892 x 10^{-8}
with MTBF	= 17,728 rounds	$\hat{\mu}$ = 10,644 rounds
Tube		
Shape parameter	= 1.995004	$\hat{\alpha}$ = 2.833641
Scale parameter	= .2712387 x 10^{-7}	$\hat{\lambda}$ = .1150474 x 10^{-10}
with MTBF	= 5,164 rounds	$\hat{\mu}$ = 6,371 rounds
Breech		
Shape parameter	= 1.911326	-
Scale parameter	= .7511622 x 10^{-9}	-
with MTBF	= 5,325 rounds	-
Reliability		
Shape parameter	= 1	α = 1
Scale parameter	= .26666 x 10^{-3}	$\hat{\lambda}$ = .28058 x 10^{-3}
with MTBF	= 3,750 rounds	$\hat{\mu}$ = 3,564 rounds

Consider the following as the requirements for the test

	Carriage	Recoil	Tube	Breech	Reliability
D_1	11,000	8,000	5,000	15,000	
D_2	22,500	22,500	7,500	22,500	
R_1					1,790
R_2					3,795

then based on the test results the following decision would be made.

Durability

Carriage - Accept - Case 9
Recoil - Accept - Case 5
Tube - Accept - Case 5
Breech - Accept - Case 3
Reliability - Fixup - Case 6

RECOMMENDATIONS

In accordance with the definitions prescribed within this report, the following table outlines the "optimized" results of the simulation.

TABLE 3

Test Description

N_1 - Number of Prototypes to be put on Test - 3
N_2 - Number of Spare Recoils/Prototypes - 5
N_3 - Number of Spare Tubes/Prototypes - 13
N_4 - Number of Spare Breeches/Prototypes - 3
Max Number of Rounds to be Fired/Prototypes - 20,000
Confidence Level for Test of Hypothesis - 90%

Requirements

Carriage	D_1	-	13,500	Breech	D_1	-	7,500
	D_2	-	21,000		D_2	-	16,000
Recoil	D_1	-	6,000	Reliability	R_1	-	400
	D_2	-	10,500		R_2	-	1,500
Tube	D_1	-	*				
	D_2	-	*				

*No Recommendation, See Section "Sensitivity and Conclusions

With the above test description and requirements, the expected total test cost is \$6,423,010.80 which can be broken down into \$3,751,500 for prototype cost and \$2,671,510.80 for ammunition. Other expected values associated with the simulated test are shown below in Table 4.

TABLE 4

Sample Size = 500

E[N] Carriage Failures During the Test = .948
E[N] Recoil Failures/Carriage = 5.68
E[N] Tube Replacements/Carriage = 8.494
E[N] Breech Failures/Carriage = .144

Number of Occurrences for Each Case*

Case No.	1	2	3	4	5	6	7	8	9
Carriage Durability	50	2	15	1	0	27	0	2	403
Recoil Durability	149	0	10	11	1	[illegible]	2	4	268
Tube Durability	-	-	-	-	-		-	-	-
Breech Durability	0	0	0	6	1	[illegible]	1	0	365
System Reliability	0	0	0	0	0	136	0	2	250

*See Section "Loss Function" for definition and explanation of each case.

Probability of Not Rejecting System at DT/OT-II

Carriage Durability	-	87.6%
Recoil Durability	-	71.4%
Tube Durability	-	-
Breech Durability	-	99.0%
System Reliability	-	100.0%
TOTAL SYSTEM		61%

Expected Total 20 year life cycle cost = \$6,223,908,800.00

SENSITIVITY AND CONCLUSIONS

A sensitivity analysis was conducted by varying the input probability distribution for each subsystem and system reliability as outlined in the section "Quantification of Performance Uncertainties." The difference in the total 20 year life cycle cost as compared to the "optimized case" are shown below.

Subsystem	Direction	Difference ($\$ \times 10^6$)
Carriage	Pessimistic	+ .991
	Optimistic	- 2.789
Recoil	Pessimistic	+ 2.6414
	Optimistic	- 2.7834
Tube	Pessimistic	+ 31.9099
	Optimistic	- 6.5098
Breech	Pessimistic	+ 5.1265
	Optimistic	- 2.3168
Reliability	Pessimistic	+ .6303
	Optimistic	- 7.2014

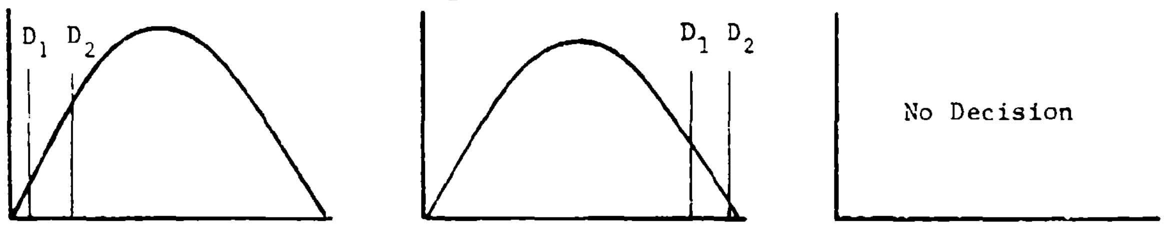
The estimate of the standard deviation σ for the total life cycle cost, due to random occurrence is $\sim \$1.3846 \times 10^6$, therefore $2\sigma = 2.7692$ and $3\sigma = 4.1538 \times 10^6$.

Since the tube showed the highest variability and was considerably outside the 3σ range it was decided to further study the tube durability requirements. Holding all other parameters at the "optimize" values, the parameters D_1 and D_2 for the tube were varied with the following results.

	Life Cycle Cost	Probability of Acceptance/ Without Redesign
$D_1 = D_2 = 1{,}000$	6.647478×10^9	100%
$D_1 = D_2 = 3{,}000$	6.313495×10^9	99.9%
$D_1 = D_2 = 5{,}000$	6.254015×10^9	95.6%
$D_1 = D_2 = 7{,}500$	6.243281×10^9	59.2%
$D_1 = D_2 = 10{,}000$	6.238615×10^9	27.6%

In each of the above outcomes, the life cycle cost was based on replacing the tube at D_2 rounds. Since there was almost no risk associated with building a tube that would last 1,000 or 3,000 rounds and the difference in total life cycle cost is above $300 million there is no reasons not to demand the 3,000 round tube. Similarly, a $59 million savings can be expected with only a 4% probability of rejection increase by requiring a 5,000 round tube. As the durability requirement is increased to 7,500 and 10,000 the percentage of savings vs. the increased probability of rejection makes one question the advisability of demanding these higher requirements. Since the simulation considers a $1 savings just as important as a $1 billion dollar savings in its effort to optimize and in addition it was assumed that the state-of-the-art was no barrier; the simulation forced the recommended durability values for the tube to the upper boundary set in the simulation. Realizing that the state-of-the-art would be a barrier at

these high levels one additional sensitivity run was made. The program test logic was changed to ignore any tube requirements and the life cycle cost was calculated based on replacing the tube at whatever wearout life could be designed for each iteration. (This would be similar to using pull-over guages.) This resulted in a total life cycle cost of 6.2239×10^9 which was even less when D_2 = 10,000 rounds. In an effort to explain this outcome consider the following three cases.



Case 1 Case 2 Case 3

The curve represents the prior probability density of the expected tube durability parameter. D_1 and D_2 define the acceptance, fix rejection region defined earlier (See Loss Function). Assume the probability density curve is the same for all three cases.

In Case 1 the rejection region is inconsequential in contribution to the expected loss. The acceptance region is large, but the longer durability life is not considered as tube replacements are based on the acceptance requirement D_2 (See Loss Function Case 9). In Case 2, the acceptance region is inconsequential. The rejection region is high in probability causing frequent rejection of the system with resulting expenditures in development of a system that meets the specified requirements, D_2, for all subsystems; and additional testing funds to validate these requirements.

Case 2 was preferred to Case 1 as the additional expenditures produce high durability while much of the predicted durability would not be utilized under Case 1.

Case 3 was preferred to Case 2: Again $D > D_2$ occurs with small probability. $D_1 < D < D_2$ results in expenditures which are approximately the same for Cases 2 and 3. If $D < D_1$ then Case 3 replaces tubes based on test estimates of durability avoiding the waste incurred by Case 1 and the expense of a new development program recommended in Case 2.

These recommendations are sensitive to the predicted estimates on tube durability. A pessimistic prediction of tube wear leads to a recommendation of Case 2 over Case 3.

The conclusions of this analysis are:

1) Expected loss is highly dependent on tube durability.
2) Sufficient tube testing should be performed to establish tube durability rather than base replacements on requirements.
3) Attainment of higher tube life is a basis of rejecting the program according to the logic of the simulation. A more realistic action would be initiation of a program to achieve a state-of-the-art tube.

4) A study should be initiated after test to evaluate the durability of the tube compared to the state-of-the-art. A study similar to this should be performed to determine benefits to be derived from accepting the tested tube design or, alternatively proceeding with a tube development program.

5) No decision regarding a tube durability requirement should be made at this time in view of the sensitivity of this parameter.

For the carriage, recoil and breech durability the "D" values as shown in the "RECOMMENDATIONS" section represent the recommended design goals or acceptance for each subsystem. The sensitivity analysis conducted by varying the prior probability distribution for each subsystem show that if the designers risk profile were in error up to 25% in either direction, the difference in the total expected loss is still close to the variability of the simulation and therefore the results are not overly sensitive to these inputs within the ±25% bands. The analysis of the simulation indicates that the reduced maintenance/replacement cost that would result by raising the D_2 values does not offset the expected increase in loss due to the increased probability of system rejection and the associated redesign-retest and related cost.

The D_1 values represent the minimum acceptable durability values. Any subsystem design which falls below these values should be rejected. Below these points the combination of redesign cost, retest cost, probability of rejection, and cost of continuing the present system are favorable as compared to the increased maintenance/replacement cost that would be incurred by fielding a weapon system with these low values.

For system reliability the R_2 value represents the design goals and the reliability value at which the system should be fielded. The R_1 value represents the lowest value for which it would be advantageous to enter into a reliability growth program and grow the system reliability to R_2. (This is based on a "Duane" growth model with a slope of .523.) An analysis of the simulation indicates that this growth slope is extremely optimistic and that a more realistic growth model needs to be developed before any recommendation can be made on the value for R_1. If the system reliability falls below R_1 then the system should be rejected and a complete redesign effort should be initiated. Until a more realistic growth model can be incorporated into the simulation, it is recommended that reliability level presently exhibited by the M102, 105MM Howitzer system be used for R_1, i.e., 400 rounds.